Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

RECENT PUBLICATIONS.

REVIEWS.

Geschichte der Mathematik in systematischer Darstellung mit besonderer Berücksichtigung der Fachwörter. By J. TROPFKE. Berlin and Leipzig, Vereinigung Wissenschaftlicher Verleger. Erster Band,[1] 1921, *Rechnen,* 8 + 177 pages; zweiter Band, 1921, *Allgemeine Arithmetik,* 4 + 221 pages; dritter Band, *Proportionen, Gleichungen,* 1922, 4 + 151 pages. Price \$.70 + \$2.00 + \$1.44.

Tropfke's *Geschichte* appeared first in 1902, as a two-volume work. It was well received as an interesting and worthy attempt at a systematic history of the fundamental ideas of arithmetic, algebra, geometry, and trigonometry.

The outstanding defect of the first edition was the reliance placed upon the authority of Cantor. This standpoint has been entirely changed in the present edition and the author has made every effort to found his conclusions upon source material. The preface indicates that the author has had the great advantage of the friendly criticism of Gustav Eneström, of Stockholm, and of H. Wieleitner, of Augsburg; no abler critics for work on the history of elementary mathematics could be found anywhere. Equally fortunate has Dr. Tropfke been in securing the aid of Julius Ruska, of Heidelberg, for points on Arabic mathematics and terminology.

The radical nature of the revision is only partly indicated by the fact that the first volume, *Rechnen* and *Algebra,* of the first edition required 332 pages, as opposed to a total of 549 pages now for these three sections, containing the corresponding material. More significant is the fact that the first edition of these sections included a grand total of slightly over 1200 footnotes, as opposed to over 2600 in the revised edition. Frequently, even yet, Tropfke cites Cantor when the original is easily available, as in respect to Egyptian duplication (note 251, volume 1) and in other places. The statement, page 52, volume 1, that the works of Jordanus had the widest circulation because of the cultivation of science largely by church schools is not all in accordance with historical fact. The works of Sacrobosco and Alexander de Villa Dei had the widest circulation, evidenced by the great number of manuscripts in European libraries, and these works also had the most enduring influence on subsequent works before the invention of printing; this conclusion has been definitely established in a thesis at Michigan, *A comparative study of the early treatises introducing into Europe the Hindu art of reckoning* (Concord, N. H., 1916) by Professor Suzan R. Benedict, of Smith College.

I note on page 13, volume 1, that the reference to the use of "Zeyfferzale" is incorrectly transcribed from the 1514 edition of Koebel's *Rechenbüchlein.* However, this is evidently only a slip as other citations checked are found accurately given.

The topics treated in the first part, of five parts planned, embrace numbers in general, including names and symbols, measures of time, of angles, and decimal

[1] Already reviewed in this MONTHLY, *1922,* 16–17.

measures, computation with integers, denominate numbers, properties of numbers, tables, fractions, and applied arithmetic. Under computation each fundamental operation is discussed with illustrations.

A notable omission is that form of division in which the remainders are written but not the partial products, commonly called the "Austrian method" and usually associated with the additive method of subtraction. Tropfke indicates that the complete "Austrian" process appeared first about the middle of the nineteenth century, but omission of partial products is found in Clavius (1603 ed. consulted), Manelli of 1659, Dechales of 1690, Malcolm's *A New System of Arithmetick* of 1730 and Sadler's *Complete System* of 1773; this method is designated as "a danda" or as Italian.

The second part of this work treats algebra and logarithms under similar detailed divisions. This method of treatment makes the work of great value to the teacher who wishes to find the historical development of any given topic of elementary arithmetic or algebra. It is highly to be recommended for this type of use.

The treatment of the algebraic symbols is given in a satisfactory manner; further light would have been obtained by consulting an article by Dr. Suzan Benedict on "The Development of Algebraic Symbolism," written at Teachers College, using the Plimpton collection and the private collection of D. E. Smith[1]. The statement (page 16) that the abbreviation used by Regiomontanus for *minus* is to be read "ig" is not at all accurate; the abbreviation is entirely regular for *minus*, the symbol like a 9 being standard for "us" and the "m" and "n" indicated by a bar above the "i."

So far as the computation with zero is concerned Tropfke leaves the impression that the early writers on algorism neglected this. However, both of the treatises published by Boncompagni (Trattati I and II), and those of Sacrobosco and Alexander de Villa Dei expressly mention this computation with zero.

As a late illustration of a mathematician who did not believe negative roots to be possible Tropfke cites Harriot's *Artis analyticae praxis*, 1631. A whole series of later illustrations is to be found in Maseres, *Tracts on Numerical Equations*.

Repeatedly in the treatment both of arithmetic and of algebra (*e.g.*, volume 1, pages 17–18; volume 2, pages 8–12; volume 3, pages 28, 36, 50, 112) Tropfke is compelled to treat the Hindu mathematics. Unfortunately Tropfke has taken the suspicions of Mr. G. R. Kaye concerning the non-Hindu origin of many mathematical ideas for fact. Historical things must be treated historically. Definite and concrete evidence has been found of numerous contributions to arithmetical and algebraical ideas by Hindu and Arabic scholars; any revision in favor of Greek origin must be based on historical evidence, not suppositions. Until the Greek documents are found no historian has any right to postulate the existence of Greek documents, and deny credit to Hindus, to Arabs, and to Babylonians solely on the basis of some hypothetical assumptions. Square root,

[1] *Teachers College, Columbia University, Circular, Department of Mathematics*, 1906–1907; *School Science and Mathematics*, vol. 9, 1909, pp. 375–384.

numerical symbols, negative numbers, the treatment of zero, fractions, algebraic equations, large numbers, and other related ideas are all topics which lead us historically to India; the whole is self-supporting and bears internal evidence of Hindu origin of more weight than any single document; but even the documentary evidence is available in mass.

Under *Allgemeine Arithmetik*, in the second volume, Tropfke treats in the first subdivision the history of our algebraic symbols and the introduction of general coefficients. The word algebra is discussed historically in a separate chapter. In a section devoted to the development of the concept of algebraic number Tropfke treats unity, zero, infinity, fractions, irrationals, negative numbers, and complex numbers. In the section devoted to the algebraic operations, the fundamental operations and also powers and roots are historically discussed. The second volume closes with a discussion of logarithms.

The historical development of topics connected with the theory of equations and of proportion is ably given in the third volume. The most notable omission seems to the writer of this review to be any mention in the discussion of quadratic equations of the algebra of Abu Kamil and of its influence upon the work of Leonardo of Pisa and Al-Karkhi [See my article in this MONTHLY (*1914*, 37–48), "The Algebra of Abu Kamil"; also in *Bibliotheca Mathematica*, volume 12, 1912, pp. 40–55]. To say that Al Karkhi followed only Greek models is nonsense; his work on quadratics is directly taken from Abu Kamil as I have shown. The date of Diophantos, 360 A.D., is too late by at least a century. The reference (page 42) to Greek normal forms of equations (*e.g.*, $x^2 = ax$, etc.) should be to Arabic forms.

The valuable list of problems and illustrations from classical authors has been increased, and an interesting appendix added on the Arabic treatment of the cubic equation.

Fortunately Tropfke has avoided, in general, the offensive national emphasis which so frequently mars the monumental work of Cantor. We see traces of Cantor's influence still in the references to Bürgi under decimal fractions in the first volume and under logarithms in the second volume.

This new edition is to be highly commended. Unfortunately no similar work is to be had in English, although even a translation of this work would be desirable. The scientific activity in Germany under the present undoubtedly distressing economic conditions arouses our admiration.

The prices in marks of books published in Germany vary almost from day to day; the publishers have affixed the prices indicated above. Notwithstanding the resentment which Americans may feel in the fact that these and other publishers in Germany make American prices higher than European it must in fairness be admitted that these prices are far less than recent English prices on volumes of this size. It is to be hoped that definite encouragement of this scholarly publication will be forthcoming from America, even though some of the orders go through German dealers to take advantage of the lower price in marks in Germany.

LOUIS C. KARPINSKI.